

# On the Size-Dependence of the Inclination Distribution of the Main Kuiper Belt

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## ABSTRACT

We present a new analysis of the currently available orbital elements for the known Kuiper belt objects. In the non-resonant, main Kuiper belt we find a statistically significant relationship between an object's absolute magnitude ( $H$ ) and its inclination ( $i$ ). Objects with  $H < 6.5$  (i.e. radii  $\gtrsim 170$  km for a 4% albedo) have higher inclinations than those with  $H > 6.5$  (radii  $\lesssim 170$  km). We have shown that this relationship is not caused by any obvious observational bias. We argue that the main Kuiper belt consists of the superposition of two distinct distributions. One is dynamically hot with inclinations as large as  $\sim 35^\circ$  and absolute magnitudes as bright as 4.5; the other is dynamically cold with  $i \lesssim 5^\circ$  and  $H > 6.5$ . The dynamically cold population is most likely dynamically primordial. We speculate on the potential causes of this relationship.

*Subject headings:* solar system: general, Kuiper Belt, formation

## 1. Introduction

The discovery of the Kuiper belt in 1992 (Jewitt & Luu 1993) issued in a new era for the study of the outer solar system. The Kuiper belt is important not only because it is a rich, new region of the solar system to be explored, but because it contains important fossil clues about the formation of the outer solar system in particular, and about planet formation in general.

Since its discovery, the Kuiper belt has supplied us with surprise after surprise. For example, before it was discovered, theorists believed that the Kuiper belt would consist of objects on low-inclination, nearly-circular orbits beyond the orbit of Neptune (Levison & Duncan 1993; Holman & Wisdom 1993). This belief seemed to be confirmed with the discovery of the first two Kuiper Belt Objects (hereafter KBOs), 1992 QB<sub>1</sub> and 1993 FW. However, the next four objects discovered revealed a real surprise. At the time of discovery their heliocentric distances were close enough to Neptune’s orbit that their orbits should be unstable, unless protected by some dynamical mechanism. Indeed, many believed that they might have been Neptunian Trojans. However, these were the first discoveries of an unexpected population of objects on highly eccentric (up to 0.3) orbits in the 2:3 mean motion resonance with Neptune (co-orbiting with Pluto).

Currently, objects in the trans-Neptunian region are divided into two main groups (see Malhotra et al. 2000 for a review). The *Kuiper belt* consists of objects that are primarily on long-lived orbits, while the *scattered disk* consists of objects that have suffered a close encounter with Neptune (Duncan & Levison 1997; Luu et al. 1997). The Kuiper belt itself is typically subdivided into two populations. Inside of roughly 42 AU, objects tend to be locked into mean motion resonances with Neptune. Most known objects in this class are in Neptune’s 2:3 mean motion resonance. However, a fraction also reside in the 3:5 and the 3:4 resonances. The orbits of all these objects are probably a result of resonance capture during the slow outward migration of Neptune during the late stages of planet formation (Malhotra 1995).

Beyond 42 AU, although several objects are believed to be in the 1:2 mean motion resonance (Marsden 2000a), most objects are not on resonant orbits. These non-resonant objects are members of what has come to be called the *main Kuiper belt*. Models of planetary migration (e.g. Malhotra 1995; Holman 1995; Hahn & Malhotra 1999) predict that unlike the KBOs in mean motion resonances, main KBOs should be on relatively low-inclination, nearly-circular orbits. However, recent observations have shown that this is not the case.

Numerous objects in this region have very large inclinations<sup>1</sup>, certainly up to about  $32^\circ$ , and most likely even higher (Marsden 2000a).

Several papers have been published which attempt, among other things, to explain the high inclinations seen in the main Kuiper belt. The mechanisms invoked to date involve the scattering of KBOs by large objects temporarily evolving through the region. It takes a massive object to excite KBOs to high inclination; much more massive than the KBOs themselves<sup>2</sup>. Petit et al. (1999) suggest that the dynamically excited Kuiper belt is caused by the passage of Earth-mass objects through that region of the solar system. Thommes et al. (1999) suggest that the large inclinations are due to the passage of Uranus and/or Neptune through the Kuiper belt while on eccentric orbits, after these planets were ejected from the region between Jupiter and Saturn. Ida et al. (2000) suggest that the Kuiper belt was excited by a passing star.

In this paper we present an analysis of the currently available orbital data of main belt KBOs which shows a new and surprising trend — an unexpected and intriguing correlation between inclination and absolute magnitude. In particular intrinsically bright objects tend to be found on larger inclinations than do intrinsically faint objects. In §2 we present the data and discuss the statistical significance of this trend. In §3 we investigate whether this trend is a result of observational selection effects. Our preliminary interpretation of this trend is presented in §4. We summarize our findings in §5.

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<sup>1</sup>Eccentricities are not a good measure of how excited the Kuiper belt is since most large eccentricity orbits are removed through close encounters with Neptune, truncating the eccentricity distribution. Inclinations do not suffer from this problem (Duncan, et al. 1995).

<sup>2</sup>A simple calculation based on an object’s escape velocity shows that it must be larger than roughly twice the radius of Pluto to scatter a Kuiper belt object to an inclination of  $30^\circ$ .

## 2. Observations

The KBO orbital elements we employ here were taken from the Minor Planet Center’s web site (<http://cfa-www.harvard.edu/cfa/ps/lists/TNOs.html> for October 20, 2000; Marsden 2000a). Before we describe our results, however, we first caution the reader about the use of such data. Although the orbital elements in this dataset are given to several significant figures, many of them are uncertain, and significant changes for individual objects routinely occur as more data is collected. This is particularly severe for objects that have been observed for only one season (B. Marsden, pers. comm.). Thus, we restrict our analysis to objects that have been observed over multiple oppositions. There are 124 such objects in our dataset; roughly a third of the total.

In general, the inclination,  $i$ , is the best determined of the 6 orbital elements because it is uniquely determined by the motion of KBO perpendicular to the ecliptic. For an object in the ecliptic and at opposition (where most KBOs have been discovered), observations taken over even just a short period of time allow for a determination of its instantaneous heliocentric distance, but do not allow for a unique determination of the semi-major axis,  $a$ , or eccentricity,  $e$ . However, since the instantaneous heliocentric distance *is* well determined (being directly calculated from the observed rate of motion), we do have a good estimate of the object’s absolute magnitude ( $H$ ).<sup>3</sup>

It also should be noted that the MPC dataset suffers from a host of observational selection effects, including those that affect inclination. Surveys for KBOs tend to search near the ecliptic and thus there is a strong selection against objects with large inclinations. Analysis of this and other observational biases is complicated by the fact that these objects were discovered by many different observing teams using different equipment and different search methods. Thus, the observational biases and limiting magnitudes vary from object to object. This complication makes it difficult to statistically analyze the KBO orbital dataset for trends. We return to this issue in §3.

Since many objects in mean motion resonances have had their inclinations affected by these resonances, we restrict ourselves to objects in the main Kuiper belt. We define members of the main Kuiper belt as those objects with  $a > 42.5$  AU (outside Neptune’s 3:5 mean motion resonance) and  $e < 0.2$  (to avoid objects in Neptune’s 1:2 mean motion

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<sup>3</sup>In planetary science, an absolute magnitude is defined as the magnitude that an object would have if it were 1 AU from both the Sun and the Earth and seen at zero phase angle, i.e. at opposition. Such a geometry can never happen in nature, but this definition is numerically convenient.

resonance and the scattered disk; Duncan & Levison 1997)<sup>4</sup>. There are 80 objects that meet these criteria.

Figure 1 shows the inclinations of these objects as a function of their absolute magnitude. The inclinations in this figure are accurate to better than  $\pm 0.5^\circ$ , while the absolute magnitudes are accurate to about  $\pm 0.5$  magnitudes (B. Marsden, pers. comm.). Notice that this figure indicates a distinct difference in the character of the inclinations for objects that have  $H < 6.5$  compared to those with  $H > 6.5$ . In order to further illustrate this point, we provide Figure 2, which shows the cumulative inclination distribution for the two populations. We refer to the absolute magnitude boundary between these groups as  $H_{break}$ .

The natural conclusion from Figures 1 and 2 is that the inclination distribution of the intrinsically faint ( $H > 6.5$ ) objects appears to be significantly *lower* than the intrinsically bright objects. Indeed, the median inclination of the faint objects is  $2.2^\circ$ , but the median inclination of the bright objects is  $12^\circ$ . Of course, assuming that there is no systematic variation of KBOs albedos, the intrinsically bright objects represent the largest KBOs<sup>5</sup>. Thus, Figures 1 and 2 suggest that the largest of the objects in the main Kuiper belt are more dynamically excited than smaller objects. This result is surprising because the mechanisms thus far suggested for exciting the Kuiper belt (see §1) have predicted such a behavior (however see Thommes et al. 2000). Because in each of these scenarios the perturber that excites the Kuiper belt is much larger than the KBOs, the response of a KBO to the perturber should be virtually independent of its size.

Before we discuss our interpretation of our new result, we first wish to demonstrate that this finding is statistically significant. After all, there are only 8 objects in our sample with  $H < 6.5$ , so in principle, small number statistics could be responsible for this result. In order to address this issue we employ the Kolmogorov-Smirnov (K-S) statistical test (Press et al. 1992), which calculates the probability that two distributions are derived from the same parent distribution, where a zero probability means the distributions are dissimilar, and unit probability means they are the same. We find that the K-S probability of the two inclination distributions seen in Figure 2 is 0.03. Thus, it is unlikely that the two distributions are the same<sup>6</sup>, and we can rule out that the two populations are the same at the 97% confidence

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<sup>4</sup>The results presented below are not significantly sensitive to our choice of these limits. For example, if we included objects with  $a > 41.5$  AU (starting inside of Neptune’s 3:5 mean motion resonance) and  $e < 0.25$ , we include 10 more objects in our sample, but neither our qualitative arguments nor our qualitative measures of statistical significance change noticeably.

<sup>5</sup>If  $p = 0.04$  then  $H = 6.5$  implies an object with a radius of  $\sim 170$  km.

<sup>6</sup>The data in Figure 2 consists of multiple opposition objects only. If we repeat this analysis using all 211

level.

We must also be careful so as to not fortuitously choose a value of the transition absolute magnitude,  $H_{break}$  (set to 6.5 above), which happens to give a low value of the K-S probability. So, in Figure 3 we present the K-S probability as a function of  $H_{break}$ . This figure shows that the K-S probability is small for all values of  $H_{break} < 6.5$ , but becomes large for values fainter than this. This result can be understood by considering Figure 1. If  $H_{break} < 6.5$ , we have only dynamically hot objects in the bright population, and since one is only adding a few dynamically hot objects to the faint group, the inclination distribution of the two groups remain roughly unchanged. If  $H_{break} > 6.5$ , one starts adding dynamically cold faint objects to the bright group. Since the cold population far outnumbers the hot bright population, cold objects start to dominate the bright group as  $H_{break}$  becomes larger than 6.5. So, the two distributions look similar.

In short, Figure 3 shows that our choice of  $H_{break} = 6.5$  is not just fortuitous and does not lead us to a false conclusion about the statistical significance of our finding. Thus, we conclude that objects with intrinsic brightnesses greater than  $H_{break} = 6.5$  actually do have an inclination distribution that is statistically different from that of fainter objects.

Could dynamical friction or physical collisions significantly modify an inclination distribution where the large objects have higher inclinations? The response timescale (Binney & Tremaine 1987) of large KBOs to dynamical friction in a dynamically cold, ancient Kuiper belt of  $50 M_{\oplus}$  (see Stern 1996) is  $\sim 10^9$  years. However, after dynamical excitation to eccentricities and inclinations characteristic of the present-day Kuiper belt, this timescale increases to  $\gtrsim 10^{12}$  years. The lower mass of the Kuiper Belt which exists today increases this timescale to  $\gtrsim 10^{14}$  years. A second potential way of modifying inclinations is through physical collisions. However, the time required for a 100-km class KBO to impact a significant fraction of its own mass in a  $50 M_{\oplus}$  Kuiper belt is also of order  $\sim 10^9$  years. Since we estimate that both the dynamical and collisional relaxation timescales are of order 100 times longer than the time required for an excited, massive KB to erode due to collisions (Stern & Colwell 1997), one must conclude that the dynamical configuration of the ancient objects in the present-day, main Kuiper belt is a well-preserved, fossil remnant of the excitation event(s) itself.

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main Kuiper belt objects with both single and multiple opposition orbits, we find a K-S probability of 0.001. Recall that the inclination and absolute magnitude of the single opposition objects are fairly well known. The uncertainty is whether they are members of the main Kuiper belt. Also, if we broaden our definition of the main Kuiper belt to objects with  $a > 41.5$  AU and  $e < 0.25$ , we find a K-S probability of 0.04.

### 3. Regarding Potential Observational Biases

In this section we investigate whether the differences seen in the inclination distributions of the bright and faint main Kuiper belt objects could be the result of observational biases. As we described above, this is a difficult issue because these objects were discovered with a variety of instrumentation and under a variety of observing conditions. In particular, the surveys that discovered the faint objects tend to have limited sky coverage, so they would not have found the bright objects, which are rare. On the other hand, the surveys that covered the most sky have fairly bright limiting magnitudes, so they would not have discovered the faint objects. Our task is made still more difficult because many surveys remain unpublished, and the details of how these discoveries were made are unknown.

Here we investigate the only two possible observational selection effects that we could think of that could erroneously lead us to the results of the last section. First, as we described above, the faint objects tend to be discovered by different surveys than the bright objects. The probability of discovering an object of a particular inclination is a strong function of the ecliptic latitude of the discovery images. Images taken at high ecliptic latitude cannot discover low inclination objects. On the other hand, images taken at low ecliptic latitude are biased against discovering high inclination objects.

The results shown in Figures 1 and 2 could be a result of differences in the ecliptic latitude of the discovery images. For example, if the surveys that covered a large area of the sky tend to stray further from the ecliptic, we might see the type of distributions seen in Figures 1 and 2. Figure 4 shows the ecliptic latitude of the objects in our sample at the time of their discovery as a function of their absolute magnitude. This data shows that the bright objects tend to be found at the same ecliptic latitudes as the faint objects. Indeed, we performed a K-S test similar to that above using ecliptic latitude instead of inclination and found the K-S probability is larger than 0.5 for all values of  $H_{break}$ . Thus, the findings discussed in §2 cannot be explained away by discovery selection effects.

Selection effects on the recovery of objects could also in principle erroneously lead to the results obtained in §2. It is well known that the brightest KBOs attract more followup observations than the faint ones. This is because the faint objects require large telescopes on which it is difficult to obtain observing time. As such the fainter objects tend to be preferentially lost. Of the objects in the main belt discovered before the year 2000 (so there was opportunity for them to have been observed during a second opposition), all the objects with  $H < 5.5$  have been recovered, while only 36% of the objects with  $H > 7.5$  have been observed again. If, for the faint objects, there is a selection against recovering high inclination objects, then the findings of §2 could be in error. To check this possibility, Figure 5 shows the fraction of main belt KBOs fainter than 6.5 that have been recovered as a function of

their inclination. We only include those objects that have discovered before the year 2000. The error-bars represent the error in the mean; they increase in size with inclination because there are fewer high inclination objects. Note that the recovery fraction for these objects is independent of inclination. Thus, the finding that objects with  $H < 6.5$  tend to have larger inclinations than objects with  $H > 6.5$  is also *not* a result of recovery statistics.



#### 4. Interpretation

Perhaps the most natural interpretation for the data in Figure 1 is that we are seeing the superposition of two distinct populations. The first population contains dynamically hot objects with inclinations up to  $\sim 35^\circ$  and absolute magnitudes as bright as 4.5. (Of course in the future, members of this hot population that are larger and/or have higher inclinations than those currently known, may well be discovered.) The other population is a dynamically cold one with  $i \lesssim 5^\circ$  and  $H \gtrsim 6.5$  (radii  $\lesssim 170$  for albedo of 4%).

There are two lines of supporting evidence in our dataset for two distinct populations. First, so far in this discussion we have restricted ourselves to the analysis of inclinations only. However, in a dynamically isotropic system, the root-mean-square (RMS) of the eccentricities should be approximately twice the RMS of the sine of the inclinations (Lissauer & Stewart 1993). So, if our ‘dynamically cold’ population is real, the eccentricities should also be small. Indeed, eccentricities should be so small that the eccentricity distribution of this population should not be truncated by Neptune. The RMS of the sine of the inclination of objects fainter than  $H = 6.5$  and with  $i \leq 5^\circ$  is 0.039, which predicts that the RMS eccentricity should be 0.078. It is observed to be 0.076 which is in good agreement. The RMS eccentricity of the remaining main belt objects is 0.11, which is significantly larger. Thus, our dynamically cold population appears to be real.

Our interpretation is also supported by Figure 6, which is the same as Figure 2, but with the  $H > 6.5$  curve scaled so that the two curves cross at  $i = 5^\circ$ . Note that the two distributions are the same for  $i > 5^\circ$ , arguing that they are members of the same population. So, we can conclude from this that the intrinsically faint objects with  $i > 5^\circ$  are part of the same population as the intrinsically bright objects. If this interpretation is correct, then approximately 40% of the objects in our sample are part of the dynamically excited population.

As we were preparing this manuscript, two papers became available that also argue for two populations in the main Kuiper belt. Brown (2000) performed detailed modeling of the one-dimensional inclination distribution of the main Kuiper belt. Although his results are somewhat model dependent, owing to an assumed functional form for the intrinsic inclination distribution of  $\sin(i) \exp(-i^2/2\sigma^2)$ , he concludes that the main Kuiper belt is most likely composed of the superposition of two distinct populations — one dynamically hot and the other dynamically cold. The dynamically cold population is best fit by  $\sigma = 2.2^\circ$ , which is consistent with our estimate that the maximum inclination of this population is roughly  $5^\circ$ .

More convincing and relevant, however, are the recent results of Tegler & Romanishin (2000), who have studied the colors of KBOs. It has been previously shown that the

Kuiper belt and scattered disk most likely contain two distinct color populations — one that is comprised of objects that are gray in color and one in which the objects are red (Tegler & Romanishin 1998). Tegler & Romanishin (2000) found that in the main Kuiper belt, all objects on low-inclination, nearly-circular orbits are red in color, while the rest of the KBOs are a mixture of both red and gray colors (also see Marsden 2000b). The black and red dots in Figure 1 represent those objects for which Tegler & Romanishin measured a gray and red color, respectively. Tegler & Romanishin’s result seems to indicate that at least the surfaces of the dynamically cold main Kuiper belt objects are chemically distinct as a group from the rest of the KBOs.

Based on the various lines of evidence we conclude that the main Kuiper belt is a superposition of two distinct populations and that these populations consist of objects with different sizes, different dynamics, and different surface properties. We speculate that a natural explanation for this result is as follows<sup>7</sup>.

Initially the protoplanetary disk in the Uranus-Neptune region and beyond was dynamically cold with size distribution and color that varied with heliocentric distance. In particular, significant numbers of large objects ( $H < 6.5$ ) had only formed in the inner regions of the disk while few, if any, objects this large formed in the outer regions. Then a dynamically violent event cleared the inner region of the disk, dynamically scattering the inner-disk objects outward. Most of these objects were either ejected from the solar system, placed in the Oort cloud, or became members of the scattered disk. However, a few of these objects would have been deposited in the main Kuiper belt, becoming the dynamically hot population described above.

This scenario has several implications. First, it suggests that objects in the scattered disk, the dynamically hot main Kuiper belt, and perhaps in Neptune’s mean motion resonances should have similar size-distributions and physical characteristics because they were all populated with the objects initially in the inner disk. In addition, since current models of the Kuiper belt show that the cold population is likely to be dynamically stable (Duncan et al. 1995), this population should not be contributing significantly to the Centaurs. Hence, the Centaurs should also have a size-distribution and physical properties similar to the dynamically hot main Kuiper belt and its cohorts. This appears to be born out by observations. Tegler & Romanishin (2000) find that the scattered disk, the dynamically hot main Kuiper belt, the plutinos, and the Centaurs roughly have the same mixture of red and gray objects. In addition, all these regions contain objects with  $H < 6.5$ .

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<sup>7</sup>In the following scenario we are assuming that the differences in  $H$  are due to differences in size. It is possible, but less likely, that it could be due to albedo differences.

Our scenario also suggests that the dynamically cold population is a dynamically primordial population; member objects most likely formed near where they are observed and have not been significantly perturbed over the age of the solar system.<sup>8</sup> It also suggests that because the intrinsically brightest objects in this population have  $H \sim 6.5$  and other brighter (larger) objects have been found in the main Kuiper belt, that the largest object to grow in this region has  $H = 6.5$  or a radius of  $\sim 170$  km (4% albedo). This result may supply important constraints on the accretional history of this region, possibly including constraints on the solid surface density of material in the region and the date of the event(s) that dynamically excited the Kuiper belt.

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<sup>8</sup>By member objects, we specifically exclude recently created collisional shards (see e.g., Farinella et al. 2000).

## 5. Summary

We have shown that the inclination distribution of objects in the main Kuiper belt most likely varies as a function of absolute magnitude. In particular, objects intrinsically brighter than  $H = 6.5$  appear to have systematically higher inclinations than intrinsically fainter objects. There is only  $\sim 3\%$  chance that these two distributions are the same. We have shown that this result is unlikely to be caused by biases in discovery or recovery observing procedures. Therefore, although it is possible that this conclusion is a result of small number statistics, we believe that it is real. Future discoveries and followups will clearly resolve this issue. The clear implication of our result is that a main belt object’s inclination is dependent on its size.

The differences between intrinsically bright objects and the intrinsically faint objects is best seen in Figure 1. Perhaps the most natural interpretation for the data in this figure is that we are seeing the superposition of two distinct populations. The first contains a dynamically hot population (inclinations up to  $\sim 35^\circ$ ) consisting of both large and small objects (absolute magnitudes as small as 4.5 or radii up to  $\sim 330$  km for albedos of 4%). Indeed, even larger objects and/or objects with higher inclinations are likely to still be found. The other population is a dynamically cold one ( $i \lesssim 5^\circ$ ) preferentially containing smaller objects ( $H \gtrsim 6.5$  or radii  $\lesssim 170$  km for albedos of 4%).

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Fig. 1.— The inclination ( $i$ ), absolute magnitude ( $H$ ) distribution of multiple opposition objects in the main Kuiper belt as of October 20, 2000. Note that objects brighter than  $H = 6.5$  are dynamically more excited than those with  $H > 6.5$ . The red dots represent red objects for which Tegler & Romanishin (2000) measured a  $V-R > 0.6$ . The black dots represent gray objects for which they measured a  $V-R < 0.6$ . The blue dots represent objects for which they have not measured colors.

Fig. 2.— The cumulative inclination distribution for members of the main Kuiper belt with multiple opposition orbits. The population is divided into two groups. The solid curve shows only those objects fainter than  $H = 6.5$ , while the dotted curve only includes objects brighter than this. A K-S test puts the probability that these two distributions are the same at 0.03.

Fig. 3.— The K-S probability that the inclination distribution of objects brighter than  $H_{break}$  is the same as that of objects less than  $H_{break}$ . The K-S probability is small for  $H_{break} < 6.5$  indicating that the two distributions are indeed most likely different.

Fig. 4.— The ecliptic latitude, absolute magnitude ( $H$ ) distribution of multiple opposition objects in the main Kuiper belt as of October 20, 2000. The ecliptic latitude was calculated at the time of discovery. Note that there is not a significant correlation between these two parameters.

Fig. 5.— The fraction of main belt  $H > 6.5$  KBOs that have so far been recovered as a function of their inclination. We only include those objects that have had the potential for being observed on multiple oppositions. The error-bars represent the error in the mean.

Fig. 6.— The same as Figure 2 except that the  $H > 6.5$  curve is scaled so that the two curves cross at  $i = 5^\circ$ .

Figure 1 —

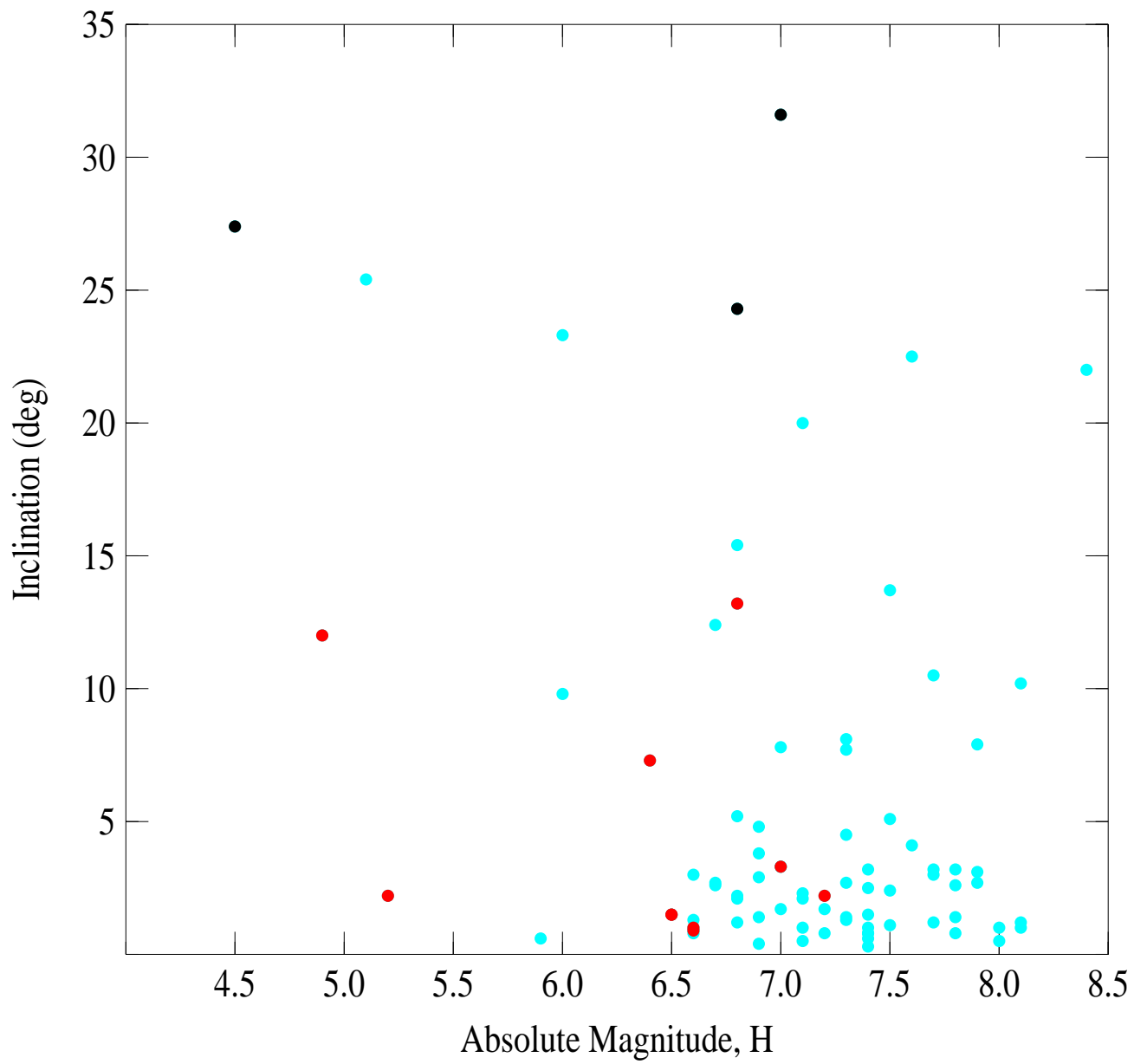




Figure 2 —

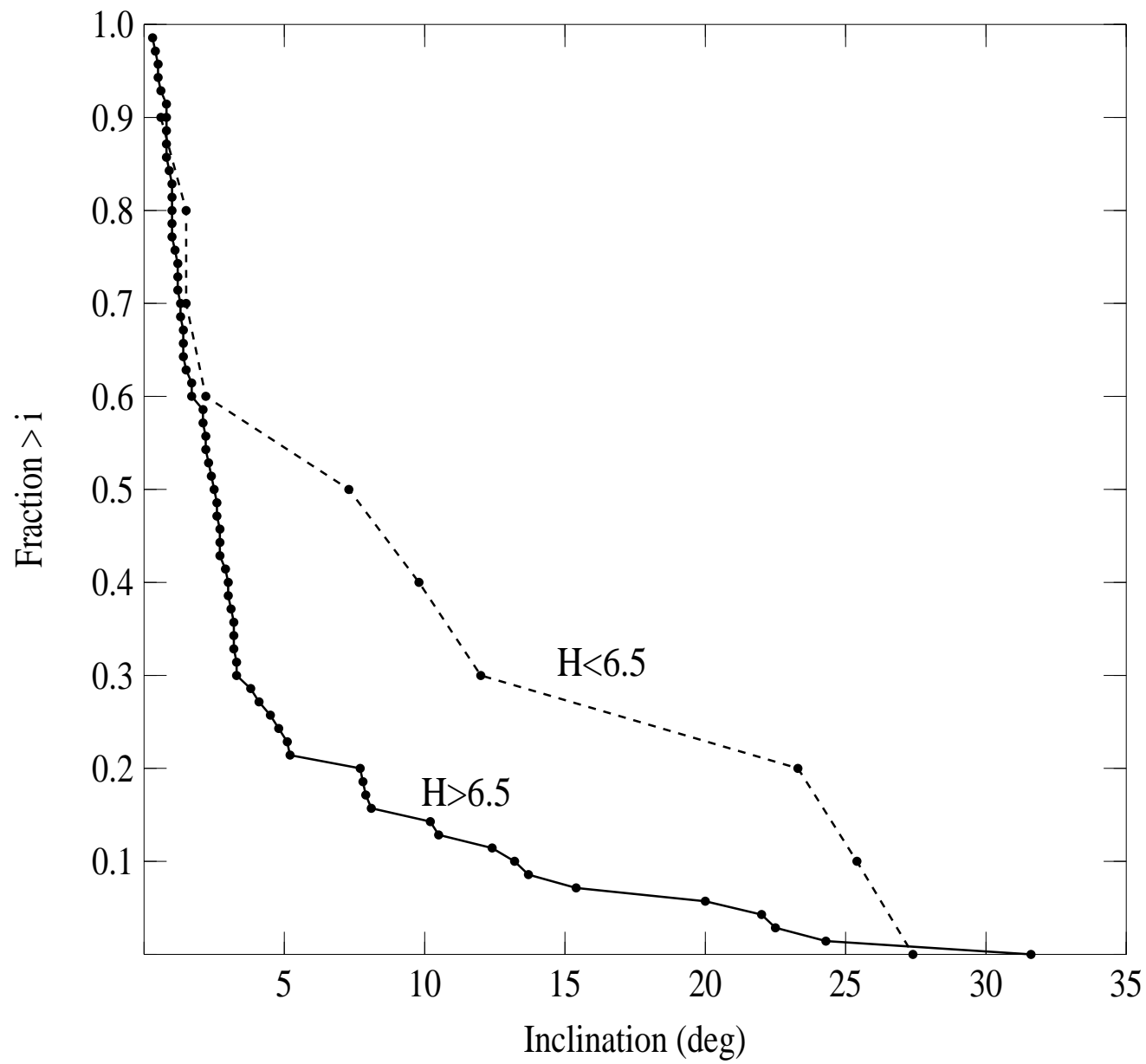


Figure 3 —

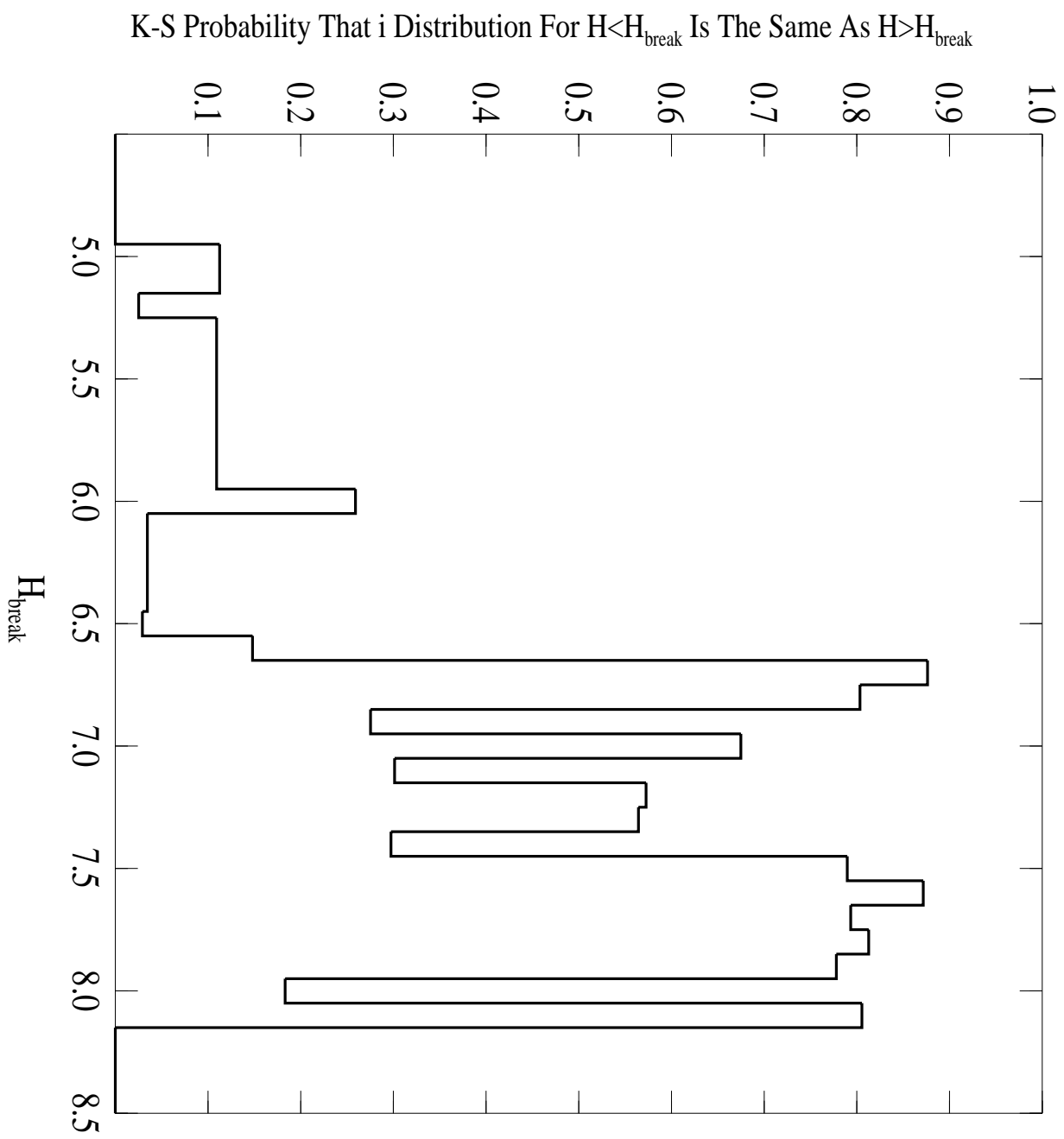


Figure 4 —

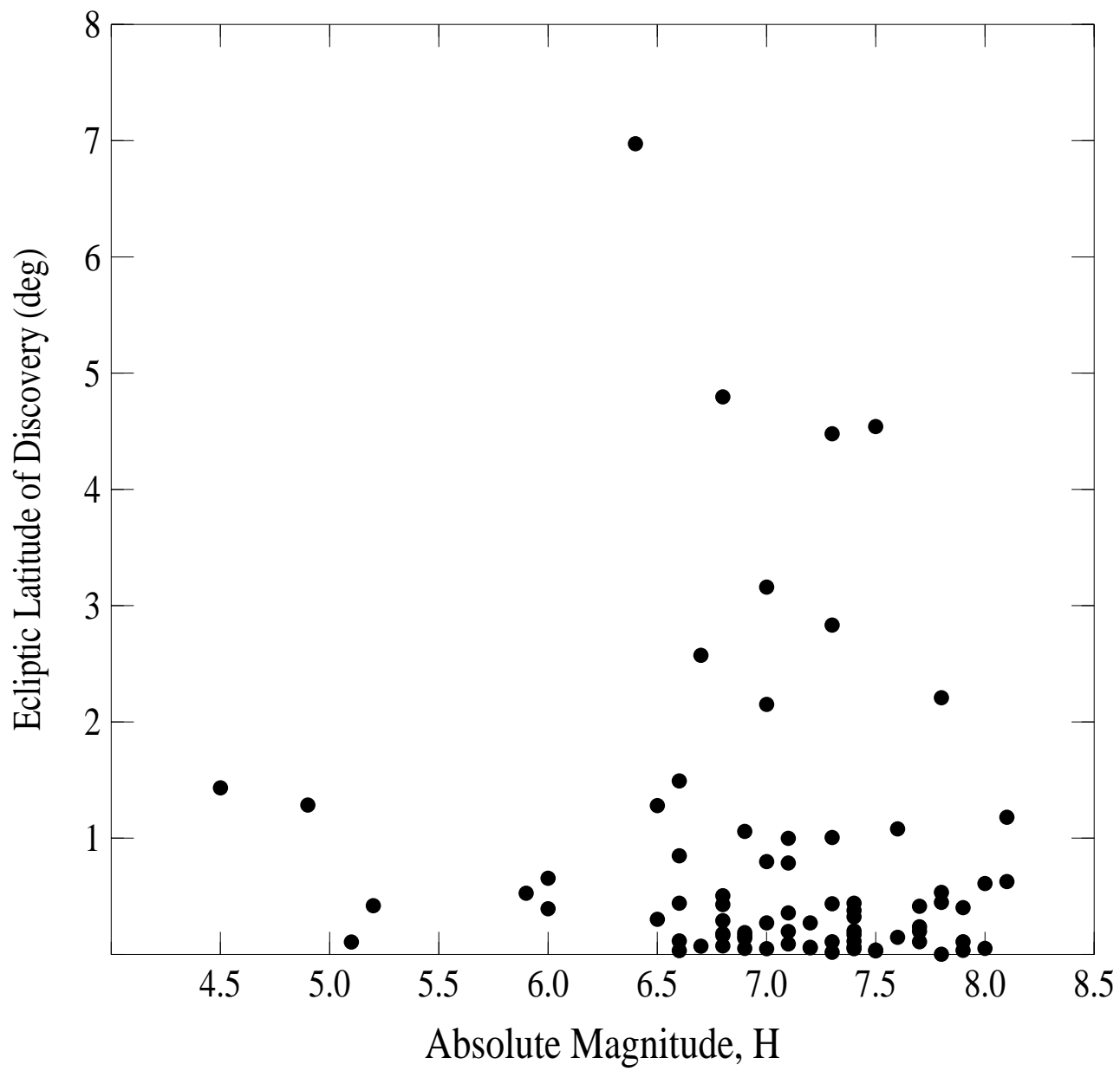


Figure 5 —

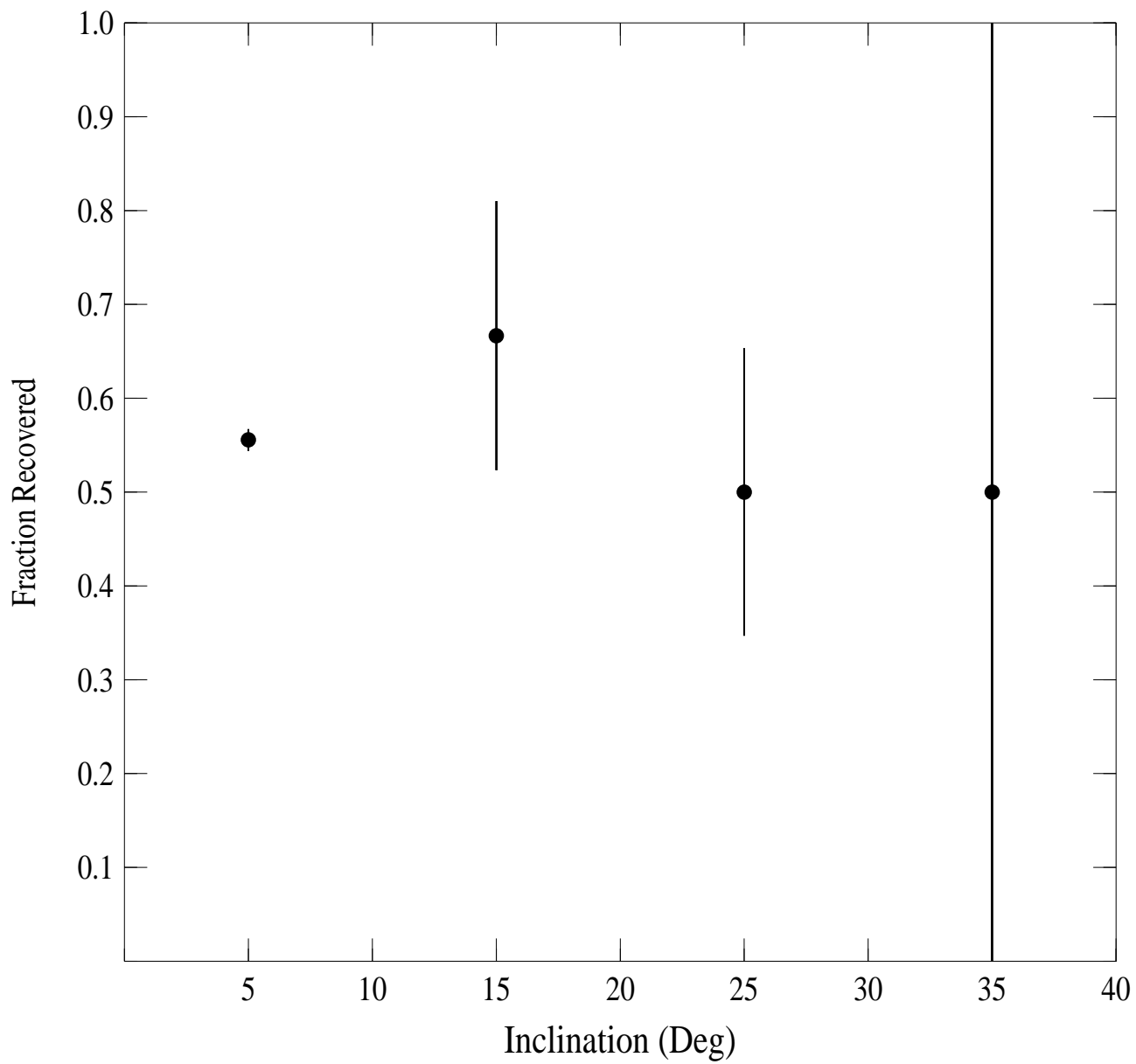


Figure 6 —

